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Active Learning with sketchometry Teaching Modules

University of Bayreuth
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Active Learning with sketchometry Teaching Modules

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sketchometry – mathematics with your index finger

The gesture-based software *sketchometry* (sketchometry.org) allows students to use their tablet or smartphone as an electronic sketchpad (see [1]). They can draw figures with their fingers that are automatically converted into accurate drawings. sketchometry is characterized by intuitive gestures that replace the usual tools of conventional dynamic math software. One significant advantage is that students can work with their own devices, so there is no need to rely on special equipment in schools.

Active learning – experimenting on a tablet or smartphone

To improve mathematics teaching and learning, we have developed and tested the concept of *sketchometry teaching modules*, which are now being successfully used in schools. These modules allow us to intensify and enhance independent, inquiry-based learning, making learning more interesting, stimulating, and sustainable for students.

The sketchometry teaching modules facilitate a step-by-step transition from traditional teaching methods to digital learning tools. It is important to emphasize that technology is not used for technology‘s sake, but rather to improve teaching and learning.

Student worksheet

Our approach is not to present the relevant content as a „ready-to-use system“. As with traditional teaching methods, students are first given a paper worksheet with instructions on what to do. Each worksheet consists of two parts. During a *construction phase*, students are instructed to draw or construct objects and configurations using sketchometry. In the subsequent *exploration phase*, students are given tasks that encourage them to experiment independently. To illustrate our concept, we consider the „triangle sum theorem“ worksheet (see below).

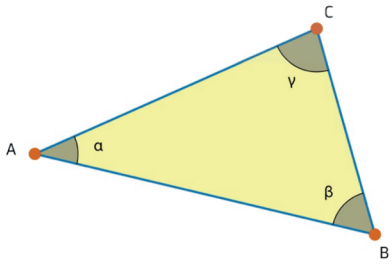
Teachers make sure that learners understand the tasks on the worksheet. Otherwise, they observe the students and intervene only, when necessary, for example by giving hints when requested. Compared to traditional teaching methods, teachers’ role has changed. Rather than primarily transferring their knowledge to the learners, they now support them in acquiring new knowledge as autonomously as possible.

Worksheet

Triangle Sum Theorem

Construction

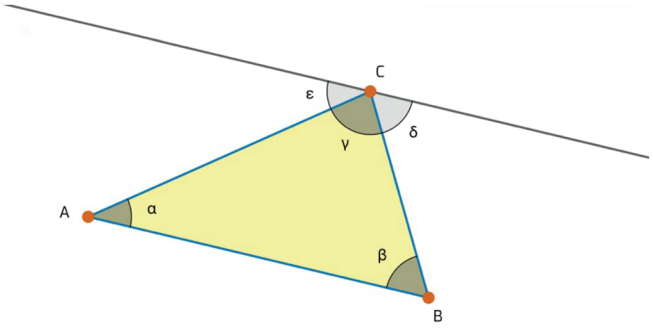
- Draw a triangle ABC and label the angles at the three vertices.



- Measure the angles and the sum of the angles. Place the measures to the right of the triangle on the board.

Exploration

- Drag any of the vertices A, B, C of the triangle.
  Observe the measures of the three angles and the sum of the measures.
- Formulate your finding as a conjecture.
- Developing a proof: Choose the strategy of adding an auxiliary line.
  Draw a line through C parallel to side $\overline{AB}$ and label the alternative interior angles.
  What is the relationship among the angles $\varepsilon, \gamma, \delta$? Try to write a proof.



Information sheet for teachers

Each student worksheet is accompanied by a teacher's information sheet. It outlines the prerequisites and objectives, indicates the level of sketchometry knowledge required, and suggests additional topics.

Information sheet for teachers

Triangle Sum Theorem

Prerequisites and Objectives

- Students are familiar with the terms triangle and angle.
- Students assume by means of a dynamic construction that the sum of the measures of the angles of a triangle is 180°.
- They prove the triangle sum conjecture by use of alternate interior angles.

sketchometry

The students should know

- how to draw a triangle,

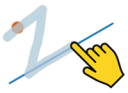
- how to tag an angle,

- how to measure angles and the sum of angles,

  Measure › *Measure* › tap an angle and place the measurement on the board

  Measure › *Calculations* › *Sum* › tap areas or measurements › tap the board at a free spot

- how to draw a line parallel to a given line segment.



Further Exploration

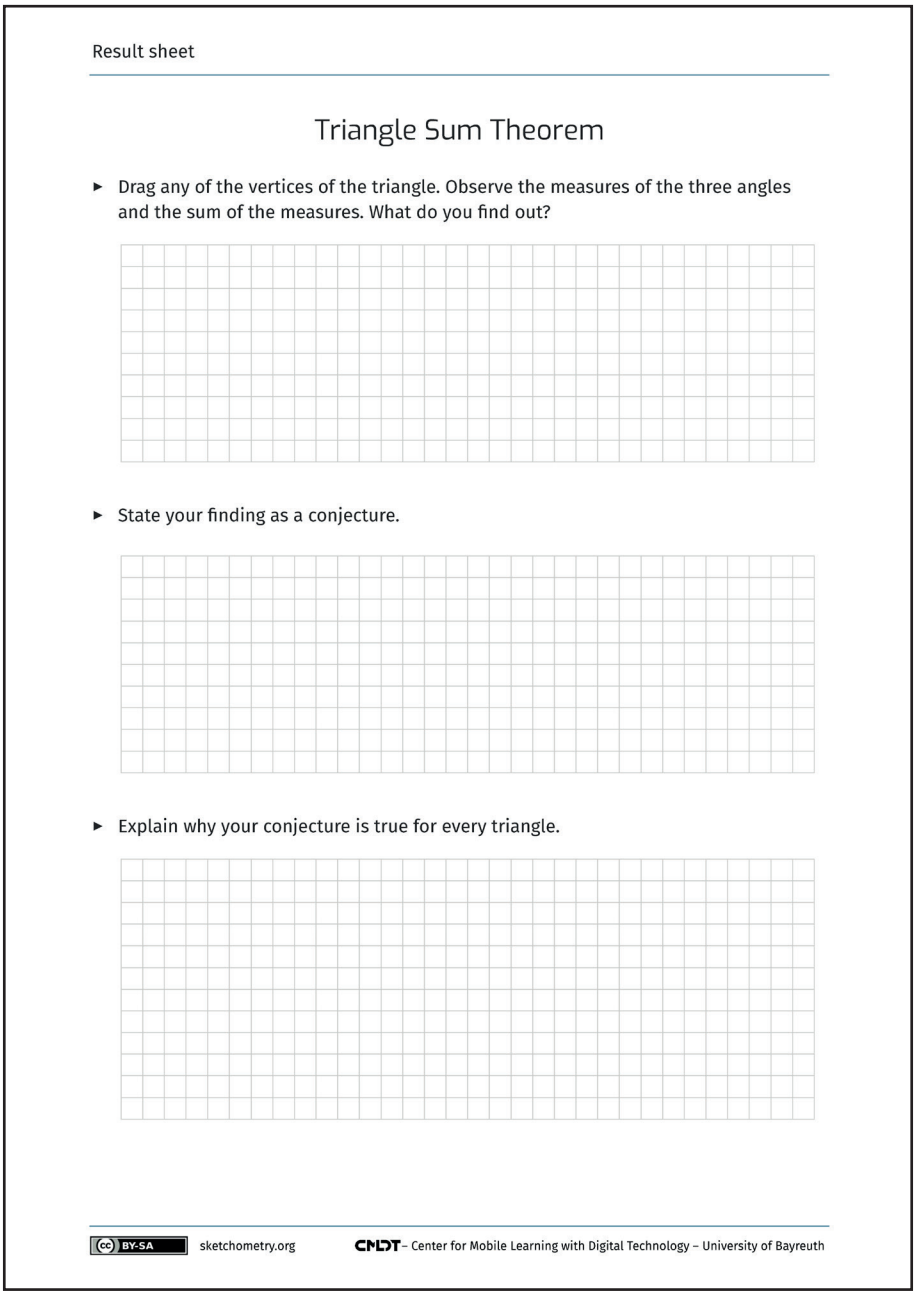
- Determine the sum of the interior angles of a quadrilateral.
  Hint: Try to use your knowledge about the triangle sum.

Documentation – recording in a study journal or on a result sheet

The „construct and explore“ approach with the sketchometry worksheets allows a way of working in the classroom that has been undervalued in mathematics. Mathematics has proven to be an experimental subject. Students can explore, observe, and experiment with constructions of their own making. But it is not enough just to do the „experiments“. To ensure that the results are not forgotten, students are encouraged to document their findings. During the exploration tasks, they are repeatedly asked to make entries by hand in a special copybook, the so-called study journal. Among other things, they should

- write down their observations,
- sketch meaningful figures,
- formulate conjectures,
- write explanations and reflections,
- record personal impressions.

To make it easier for learners to keep individual records, a result sheet can be produced for each worksheet as an alternative to the study journal. The expected entries can be clearly structured with appropriate text.
The nature and extent of the structuring guidelines will depend on the learners‘ ability to work individually. If they are experienced enough with independent working, pre-structured result sheets will not be necessary.
These individual records in a study journal or on a result sheet, together with the corresponding worksheet, document the content covered. They complement the textbook and can be used to prepare for exams.



Result sheet

Triangle Sum Theorem

- Drag any of the vertices of the triangle. Observe the measures of the three angles and the sum of the measures. What do you find out?
- State your finding as a conjecture.
- Explain why your conjecture is true for every triangle.

Discussing ideas – presenting results

After working on a worksheet individually, learners can compare and discuss their notes on the result sheet or in the study journal. This enables them to learn how to describe mathematical content in their own words and express themselves in „mathematical language“. Additionally, this active communication leads to a deeper understanding of the learning content.

During the subsequent *discussion phase*, learners present their results. With the appropriate technical infrastructure, each student's tablet contents can be displayed via a data projector. If necessary, the teacher can intervene to correct, complete, and supplement the explanations.
Using sketchometry worksheets described here promotes problem-oriented and cooperative learning. Ideally, this type of teaching and learning should be an everyday part of the lesson, not a special feature.
While learners work on the worksheet, the teacher observes the class, is available for questions and offers help for self-help. During the discussion phase, the teacher leads the presentation and discussion of the students' contributions. In a final teacher-centered *instructional phase*, the teacher summarizes the discussion and puts it in the context of the subject. Alternative approaches and arguments can be discussed, and new mathematical terms can be introduced.

Think-Pair-Share – school should enable individual learning

The teaching method presented here is based on four phases: constructing, exploring, discussing, and instruction. It is a combination of active, independent phases of work by the students (guided by instructions to construct, explore, document, and discuss) and a phase of presentation and checking of results moderated by the teacher. This approach is inspired by the „think-pair-share" learning technique.

Think phase

First, students work independently on the worksheet. They complete the construction tasks, followed by the exploration tasks. As they work, they write down their observations, assumptions, and findings on the result sheet. The teacher provides assistance if necessary or upon request.

Pair phase

Students compare their results with those of their partners or within their learning group. They add to their notes if necessary. The teacher observes each group and is available to give advice (help for self-help). This allows the teacher to find out what assumptions or solutions are being made and thus gain an overview of the results obtained.

Share phase

The results are discussed with the entire class. This can be done using a completed result sheet from a learning group. During this teacher-centered phase, the teacher guides the discussion, corrects and adds to the results as necessary. New terms can also be introduced at this stage.

Confirming the results

Together with the students, a model result sheet can be produced and made available to the entire class, either electronically or on paper. This collaborative work is preferable to a result sheet prepared by the teacher. Otherwise, there is a risk that the students will not contribute enough because they know that they will receive a model sheet from the teacher.

Exercise sheet

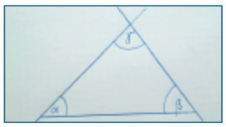
The student worksheet, the result sheet and the teacher information sheet are the basic elements of a sketchometry teaching module. Additionally, an exercise sheet with tasks to practice and/or further explore the worksheet topic (with or without sketchometry) has proven useful. Such an exercise sheet can easily be adapted or modified according to the students' level.

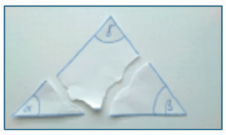
Exercise sheet

Triangle Sum Theorem

1. Draw a triangle on a sheet of paper and label the interior angles α, β, γ.
   Carefully cut out the triangle along the sides.

   Tear off the three angles.





   Put the three pieces of paper together so that the vertices meet at one point.
   What do you find?

2. In a triangle, the interior angle measures $\alpha = 37°$. Choose the other two angles so that the triangle is

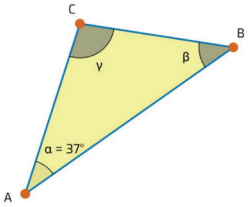
   a) acute-angled, $\beta =$ _____ $\gamma =$ _____

   b) obtuse, $\beta =$ _____ $\gamma =$ _____

   c) right-angled, $\beta =$ _____ $\gamma =$ _____

   d) isosceles, $\beta =$ _____ $\gamma =$ _____

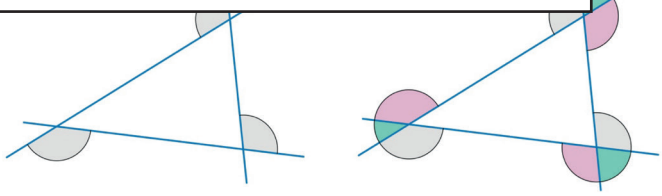
   e) equilateral. $\beta =$ _____ $\gamma =$ _____



3. Determine the sum of the angles in a

   - quadrilateral
   - pentagon,
   - hexagon,
   - n–gon.

Explain your solution in each case (with a sketch).



Student worksheets as videos

The student worksheets come to life. The videos demonstrate the construction and exploration process. They motivate the students to engage with the content. The videos are also particularly suitable for individual repetition of specific lessons and for independent learning when relevant lessons have been missed (e.g., due to illness). Videos are also necessary when working with the flipped classroom method.

Flipped classroom method

In traditional teaching methods, homework is typically assigned to practice and reinforce what was taught in class. In the flipped classroom method, however, homework can be the first step toward learning new content. Rather than absorbing knowledge passively, students must actively acquire it with the help of sketchometry worksheets. During the construction, exploration and documentation phases, students work on their own. Additionally, videos of the corresponding worksheets can be made available to promote better understanding. The presentation and the teacher-centered instruction phase take place at school. If learners are not accustomed to independent work, it is advisable to first introduce this method during student-centered work phases at school. The teacher is available as an advisor and only intervenes when necessary.

Structure of a sketchometry teaching module

The general structure of a teaching module is as follows: Each module is dedicated to a specific topic and consists of the following five elements:

1. Information sheet for teachers
   Prerequisites and objectives for the student worksheet.
   Notes on sketchometry: List of gestures needed for the worksheet.

2. Student worksheet
   Construction instructions.
   Instructions for exploration, experimentation, and documentation.

3. Result sheet
   Pre-defined structure to easily document by hand results of exploration and experimentation tasks.
   Space for individual notes and sketches.

4. Exercise sheet
   Tasks to practice or further explore the topic of the student worksheet.
   Further tasks and additional topic suggestions.

5. Video accompanying the student worksheet.

Reasons for using sketchometry teaching modules

The following list summarizes the advantages of using sketchometry teaching modules in the classroom as described above, especially compared to traditional dynamic worksheets.

- Individual construction makes the task more familiar.
- A self-made construction provides a sense of achievement. This increases motivation to work on exploratory tasks.
- Unlike a prefabricated construction, an initial construction process does not allow for quick, unreflective experimentation with dynamic configurations. Learners must engage with the topic of the worksheet.
- Work is „slowed down“ by the need for handwritten notes.
- Those who write must first reflect on what they want to write. Learners need to structure their thoughts.
- By thinking and writing, learners become more engaged with the topic. This makes the subject more sustainable.
- Learners need to be active.
- Independent work is encouraged.
- Learners can largely determine their own learning speed.
- No prior technical knowledge is required.
- No experience with mathematical software is required.

What do we achieve through innovative teaching and learning?

In recent decades, many suggestions have been made to improve the effectiveness of mathematics and science education. Numerous international and national projects have promoted and tested innovative teaching and learning methods, such as:

- inquiry-based learning,
- the use of digital media with appropriate software,
- learning in context,
- cooperative learning,
- the involvement of extracurricular learning locations.

These efforts aim to increase students‘ interest in STEM subjects to better prepare them for their future professional and private lives, which will be increasingly shaped by information and communication technologies and science and technology.
The question now is whether these efforts have been worthwhile. In addition to the obvious changes in teaching and learning, have these methods positively impacted learners‘ attitudes toward the subject? Most importantly, have they led to better performance?
Specific studies have examined individual projects, but can general statements be drawn from them? As part of a meta-analysis, researchers at the University of Utrecht evaluated 56 publications from 1988 to 2014 (see [2]). They found that innovative teaching methods improved learners‘ attitudes toward the subject and their performance, particularly regarding the above-mentioned methods. Thus, no significant differences between the different approaches were identified. For effective, sustainable teaching, teachers should choose the method that best suits the learning objective and their teaching style.

Why can we say that sketchometry teaching modules improve teaching and learning?

sketchometry teaching modules facilitate inquiry-based learning, learning in context, cooperative learning, and the use of software in the classroom. Moreover, learning with sketchometry worksheets is not limited to school. Therefore, the study also provides evidence that using sketchometry teaching modules positively impacts geometry teaching and learning.

Looking to the near future

What further impact can sketchometry teaching modules have on classroom teaching? Firstly, individual teaching modules complement the textbook. A worthwhile goal could be that a suitable collection of teaching modules would cover large areas of the school curriculum and eventually replace the traditional textbook.

References

[1] Baptist, Peter: Maths with the index finger – sketchometry. CMLDT publications, 1, University of Bayreuth, 2023 https://mobile-learning.uni-bayreuth.de/CMLDTpublications

[2] Savelsbergh Elwin R., Gjalt T. Prins et al.: Effects of innovative science and mathematics teaching on student attitudes and achievement: A meta-analytic study. Educational Research Review 19, 2016, 158 – 172.