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Maths with the index finger – sketchometry

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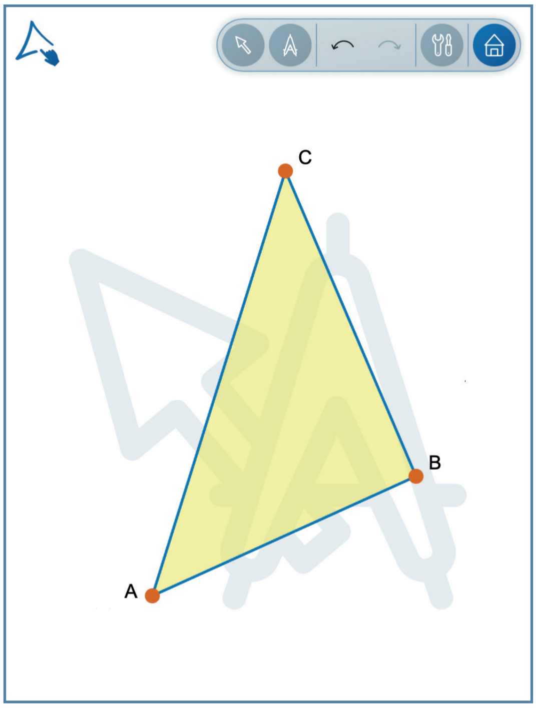
Drawing with the finger was a common method in antiquity to sketch or illustrate ideas and considerations. There was neither paper nor pencil to quickly make a sketch, instead the geometers of that time drew figures with their fingers on a smoothed sand surface. The anecdote about the tragic end of Archimedes (ca. 287 BC–212 BC) during the conquest of the Sicilian harbor city of Syracuse by the Romans is well known. While the soldiers were looting in the city, Archimedes was intensively engaged in geometrical considerations. A Roman soldier stormed into the scholar‘s house and destroyed the geometric figures he had drawn in the sand. Angrily, Archimedes complained with the words "Noli turbare circulos meos". The soldier then killed him, much to the displeasure of his general Marcellus. The commander was eager to capture Archimedes so that the Romans could profit from his engineering skills in the future.


sketches in the sand

In contrast to sketches or constructions on paper, sketches in the sand are very imprecise; they could only serve as a basis for considerations for the ancient scientists. Geometry itself took place in the mind. Therefore geometry was established as an abstract science.

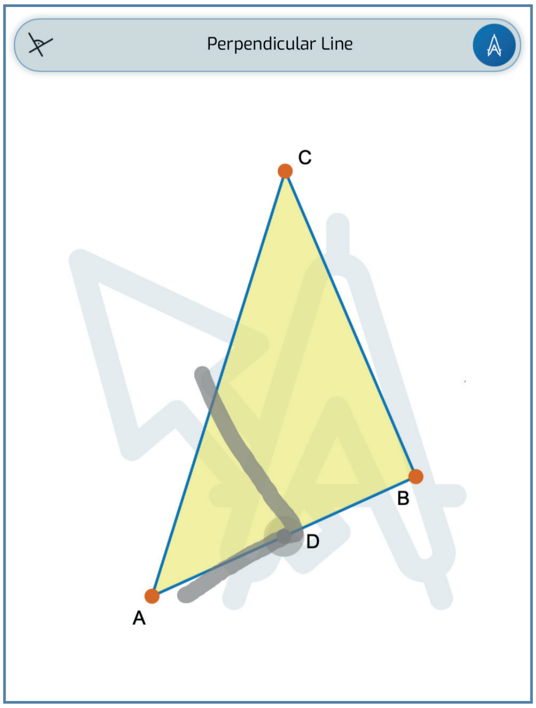
Drawing tools and materials have changed over the centuries due to technological progress. Nevertheless, drawing with the finger is being propagated again in mathematics lessons in the 21st century. However, smoothed sand is no longer used as the drawing surface. The finger sketches are performed on a tablet or smartphone screen and immediately converted into accurate figures with the help of the sketchometry software (https://sketchometry.org), as the following example of drawing a triangle shows.

sketchometry gesture

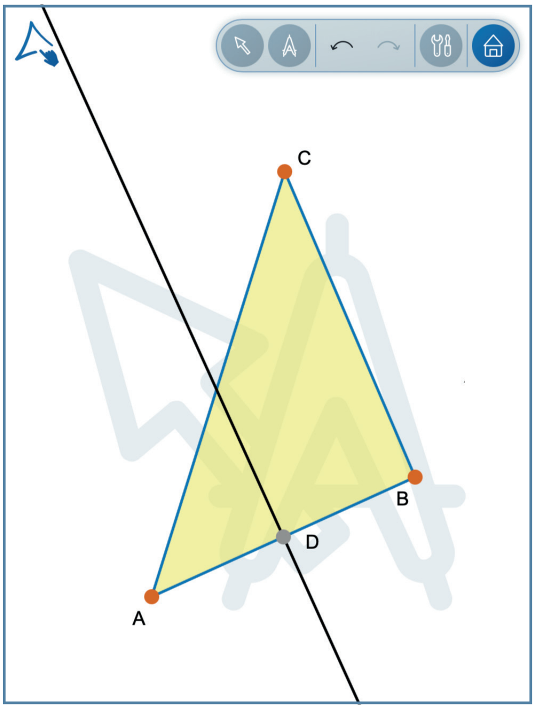


recognized triangle

Intuitive gestures, which are also performed with the index finger, are the characteristic feature of sketchometry. With the help of the perpendicular gesture, for example, the perpendicular bisector is drawn through the midpoint of the side [AB] of the triangle.



perpendicular line gesture – right angle sketch



recognized perpendicular line

In sketchometry, gestures thus replace the usual tools of traditional dynamic software. In addition, geometric objects can be "grasped" and continuously modified with one or two fingers. The learners see immediately what they are doing. Smartphones or tablets with the appropriate software have proven to be ideal sketching tools for inquiry-based learning.

Fingers have always been a popular tool for learning. For example, toddlers and children in early primary school often use their fingers to add and subtract numbers. Counting quantities is also easier for them when using a finger. In order not to forget any element, children point to the individual objects with their index finger.

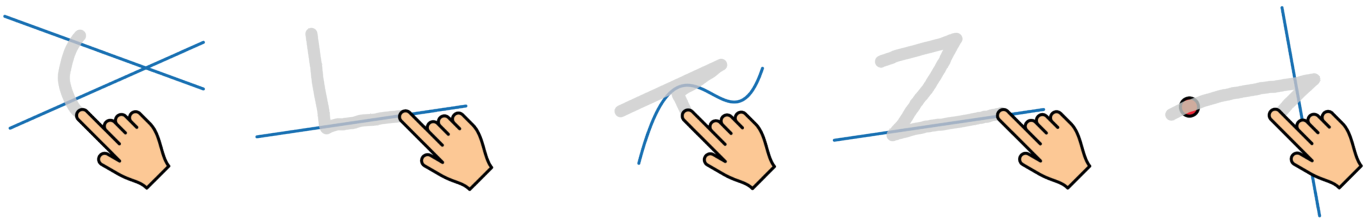
The finger technique offers even more advantages. In recent years, Australian scientists have shown in several studies that tracing figures with the index finger can help solve mathematical problems. For example, schoolchildren in Sydney between the ages of nine and thirteen had less difficulty solving geometric problems if they not only read the corresponding texts carefully but were also encouraged to trace certain elements of the problem, such as angles at intersections of straight lines, with their index finger. The results of the research group led by Paul Ginns (University of Sydney) suggest that learning mathematics can be improved if the learning process is supplemented by tracing with the finger. "Our findings have a range of implications for teachers and students alike. They show maths learning by young students may be enhanced substantially with the simple addition of instructions to finger-trace elements of maths problems," notes Ginns (cf. [3]).

The researchers assume that pointing with the finger or tracing the angles of a triangle with the index finger increases attention and thus gives this information a processing priority in the brain. Tracing with the finger could also reduce the load on working memory and improve the ability to retain more complex information better, as other brain regions are activated in addition to visual perception. "At the classroom level, teachers can assist students to learn new mathematical content by giving instructions to ‚trace over' the important elements of worked examples that already appear in mathematics textbooks or worksheets. This simple, zero-cost teaching approach can enhance the effectiveness of mathematics instruction across multiple areas of the subject," writes Ginns (see [3]). In addition, based on a study with medical students, he assumes that adults also learn more sustainably and with greater understanding if the teaching material is not only presented visually and acoustically, but if its communication is also supported by gestures (cf. [5]).

"Learning with the finger" was already propagated by Maria Montessori (1870 – 1952) at the beginning of the 20th century. To learn the alphabet, she used letters made of sandpaper. The children listen to the letters spoken by the teacher, look at their appearance and trace them with their index finger. At that time, there was no proof of the efficiency of this method; one simply relied on intuition. Only later studies proved that tracing with the finger is helpful in recognising letters and geometric shapes.

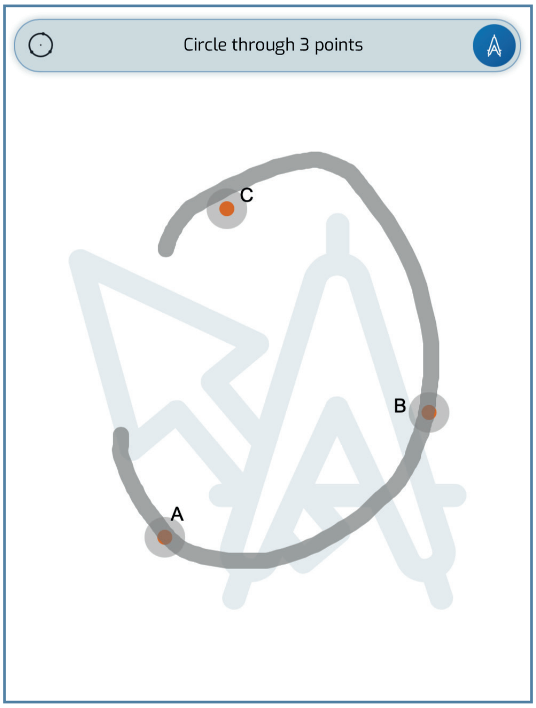
Maria Montessori (1913)

According to the latest findings, appropriate gestures can also facilitate learning a foreign language. Those who gesture with their arms in addition to traditional learning techniques such as reading, writing, listening, and repeating are better able to save vocabulary in their memory, because linking up with motoric areas of the brain strengthens word memory. This is the conclusion of a research team from the Max Planck Institute for Human Cognitive and Brain Sciences in Leipzig. In the "Journal of Neuroscience", the researchers therefore advocate the use of learning techniques that involve the body's motoric system (cf. [2]). Another team from the Max Planck Institute has found that gestures particularly promote the memory of words if they represent the meaning of the word visually. In addition to the motoric component, sensory experience of the word meaning thus also seems to play a role.
The software sketchometry uses these advantages of linking with motoric brain regions in learning and understanding mathematics. In sketchometry, gestures replace the traditional tools of dynamic geometry software. With their index finger students draw figures, which are automatically converted into an accurate representation. The sketchometry development team has made sure that these gestures are visually close to the object to be created (e.g. angle, perpendicular, tangent to a curve, parallel to a straight line, reflection on a straight line, ...). These gestures are intuitive for students and therefore easy to remember.

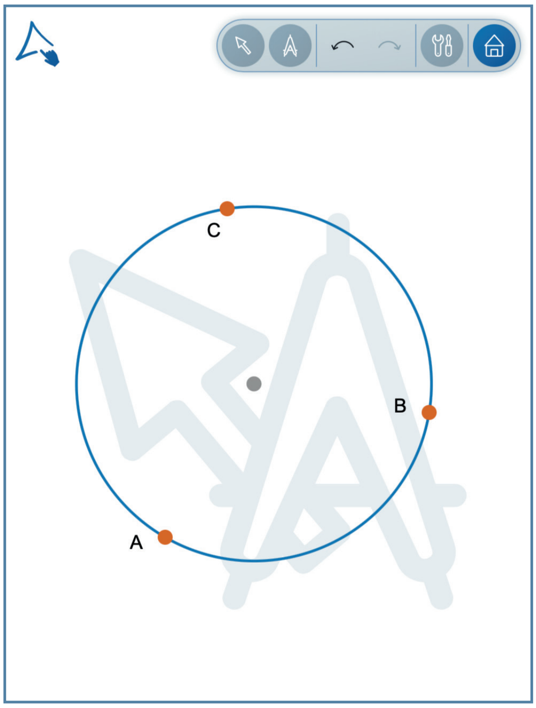


The fine-motor movements of the finger when sketching the gestures involve brain regions in the learning process that are not activated when clicking on an icon or an element of a menu list. Clicking is always the same process, regardless of which geometric activity is initiated by it. The gestures, on the other hand, differ from each other depending on the desired construction.

For example, if a circle is to be drawn through three given points, one indicates the circle with a corresponding gesture through the three points. The learners immediately make a conscious circular movement with their index finger. As an "encore", the programme creates the corresponding circle center at the same time as the circle line.

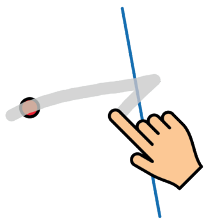


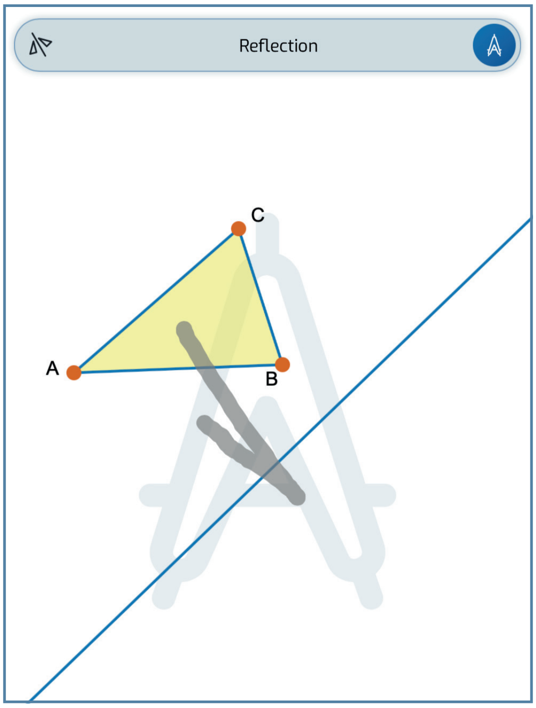
sketch of a circle through three given points



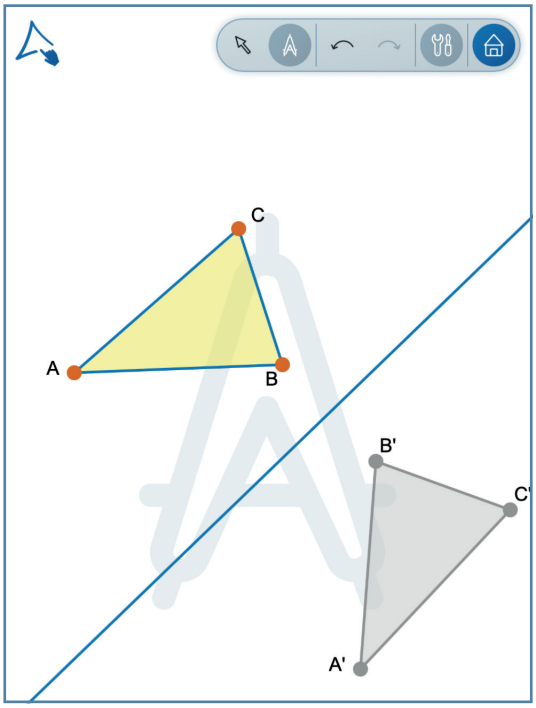
circle with center

An arrow is the gesture for reflection. The object to be reflected can be a single point, a straight line, a distance or even a figure. In the example below, a triangle is reflected on a straight line. The arrow starts in the object to be reflected; the arrowhead sweeps the axis of reflection.





reflection gesture



reflected triangle

Such gestures are the essential elements of sketchometry. The software therefore does not require a complex menu structure. This is especially advantageous on small displays. This makes sketchometry particularly suitable for use on smartphones.

The results of the Australian and Leipzig researchers suggest that the positive learning effects through linking with motor brain areas also apply to the subject of mathematics when students work with sketchometry. This is because they use the index finger to create their constructions and when applying the intuitive sketchometry gestures. The dynamic possibilities of the software are used when the created configurations are modified by dragging with the index finger to discover certain properties or relationships.

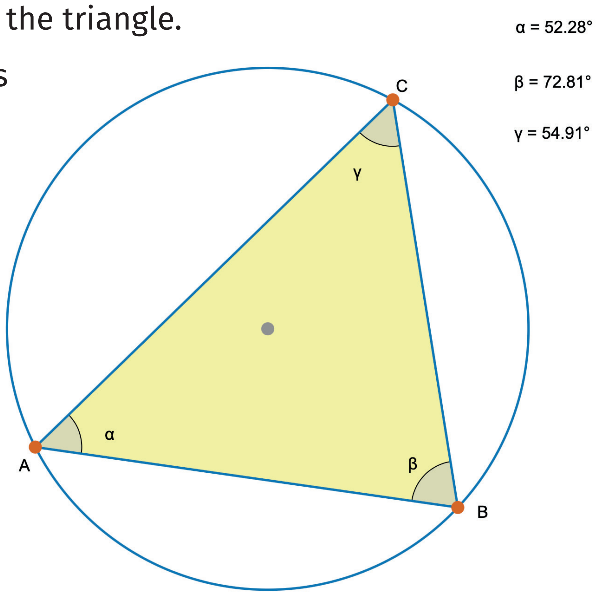
When exploring the position of the center of the circumcircle of a triangle depending on the shape of the triangle, the above-mentioned advantages of working with the index finger can be clearly seen. The corresponding worksheet is divided into the two sections: Construction and Exploration.

Worksheet

Position of the Circumcenter of a Triangle

Construction

- Draw a triangle and mark the interior angles.
- Measure the interior angles and place the measured values next to the triangle.
- Draw the circumcircle of the triangle.
  Hint: The circumcenter is created automatically.



Exploration

- Drag any of the vertices of the triangle and observe the position of the circumcenter (inside or outside of the triangle, on one side of the triangle).
- Take notes of your observations depending on the shape of the triangle (acute, obtuse, right angled).

The learners use their index finger to draw the triangle, to mark the interior angles and to determine the position of the measured values on the drawing plane, and to sketch the circle through the three vertices. Modifying the shape of the triangle is again done with the help of the index finger by touching one of the vertices with the finger and then dragging the point. All relevant construction steps and considerations are not only based on visual perception, but motor brain areas are always involved as well.

In addition to the described advantages of learning with the finger, worksheets with tasks for exploration prove to be advantageous for teaching for another reason: mathematics turns out to be an experimental subject. Because the students are not given ready-made results but are guided to go on a journey of discovery by themselves. In addition, they can determine their own learning pace to a large extent. To make it easier for the students to take individual notes during the exploration phase, a so-called results sheet can be created as a companion to each worksheet. The expected entries can be clearly structured by means of corresponding text guidelines. In class, the construction and exploration phase is followed by a joint discussion of the findings. This procedure is based on the well-established three-step approach of "Think – Pair – Share". The respective topic is concluded with a teacher-centered instruction phase in which, among other things, the contents discussed are summarised and any necessary additions are made.

With the work sheet and the results sheet, you already have the first two components of a so-called "sketchometry teaching module". The following overview shows the complete structure of such a teaching module.

Structure of a sketchometry Teaching Module

Topic of the Teaching Module

1. Information sheet for teachers
- Prerequisites and objectives for the student worksheet
- Notes on sketchometry:
  listing of the gestures needed for the worksheet

2. Student worksheet
- Instructions for constructing
- Assignments for exploring, experimenting, and documenting

3. Results sheet
- Predefined structure as a help for easy documentation of the results of the exploration and experimentation assignments
- Possibility for individual notes and sketches

4. Exercise sheet
- Tasks to practice or deepen the topic of the student worksheet
- Further tasks or additional topical suggestions
- Additional thematic suggestions

5. Video to the student worksheet

The concept of the “sketchometry teaching modules“ – divided into the phases of constructing, exploring, documenting, discussing, presenting – is intended to foster independent learning. An example of such a teaching module on the topic “triangle sum conjecture“ as well as proposals for teaching with sketchometry can be found in [1]. Over time, a collection of teaching modules can be successively created and then combined into a workbook. In this approach, tablets and smartphones prove to be a learning tool that students use for sketching and discovery. One advantage that should not be underestimated is that students can work with their own devices and are not dependent on special equipment in schools.

Conclusion: Those who prefer the index finger to the keyboard and mouse have an advantage. The innovative software sketchometry in combination with the concept of teaching modules promotes teaching and learning in mathematics.

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Photos

Sketches in the Sand (p. 2), Miller, CMLDT

Maria Montessori (p. 4), public domain, commons.wikipedia.org